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PHYSICAL AND QUANTUM OPTICS

Reflection and Transmission of Light by Cholesteric Liquid Crystal–Glass–Cholesteric Liquid Crystal and Cholesteric Liquid Crystal(1)–Cholesteric Crystal(2) Systems

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Abstract—Reflection and transmission of light by cholesteric liquid crystal–glass–cholesteric liquid crystal and cholesteric liquid crystal(1)–cholesteric liquid crystal(2) systems were studied. The classical Ambartsumyan method of adding layers and the concept of a sewing function were used. This approach was developed earlier in astrophysics for the theory of radiation transfer. Here, we used a version of this method adapted to wave optics. The Jones matrices are constructed for these systems. The features of the reflection and transmission spectra, optical rotation and ellipticity of polarization were studied for these systems. It is proposed to use these systems as tunable narrow-band filters and mirrors. These systems can be used, for example, to develop a variety of optical elements for lasers and of polarimetric elements in ellipsometry. The specific features of eigenpolarization are also discussed. It is shown that optical rotation of the two layers of cholesteric liquid crystals, which differ from each other only by the sign of the helix, is nonzero, and it becomes substantial in the diffraction reflection region. A unique property of these systems is the degeneracy (coincidence) of eigenpolarizations. *© 2000 MAIK "Nauka/Interperiodica".*

INTRODUCTION

Extensive application of liquid crystals have stimulated considerable interest in the study of optical properties of various composite structures containing liquid-crystal layers and, in particular, cholesteric liquid crystals (CLCs). The problem of normal incidence of a monochromatic plane wave on a CLC layer with the uniform helixlike structure has been adequately studied [1–3]. The results of numerical and approximate analytical studies are well known, and recently an exact analytical solution of this boundary problem was obtained [4]. It is very attractive to use this exact solution to analyze multilayer optical structures containing CLC layers.

The efficiency of the direct application of the boundary condition method for solving the problem of propagation of an electromagnetic wave through multilayer structures decreases with an increasing number of the layers in the system because the number of equations subject to analysis increases. For example, the treatment of a three-layer glass–CLC–glass system involves a numerical analysis of a system of 32 equations with 32 unknowns [5, 6]. The computer analysis of such a system is not a problem, but even in this simple case the account of absorption or large anisotropy of the medium requires considerable efforts to maintain reasonable accuracy of calculations. Therefore, there is a need to find methods and procedures where the dimension of the system of equations does not depend on the number of layers in the multilayer optical system (for example, recurrent methods or the 4×4 matrix method, etc.).

A variety of methods of "layer addition" have been long used in crystal optics to analyze multilayer structures (see [2, 7–10] and references therein). In this paper, we use a simple and elegant modification of the Ambartsumyan method of the addition of layers (invariance principle), which was proposed in [11] and developed in [9]. This method provides an exact solution in the sense that it takes into account all multiple reflections at all interfaces. It is assumed that the solution of the "reflection–transmission" problem for each layer is known beforehand.

The classical Ambartsumyan method of the addition of layers [11] and the concept of a "sewing" function [9, 12] were earlier developed for astrophysical problems of multiple scattering in turbid media. To use these results in wave optics, it is necessary to pass from a description in terms of the intensity to a more general description of optical phenomena in terms of "amplitude and phase." In this way, we studied below the transmission and reflection of light in the CLC–glass–CLC and CLC(1)–CLC(2) systems. This optical model is useful in connection with the possibility of creating artificial helical media (including media with specified parameters), as well as artificial ferromagnetic helical structures, which can imitate the properties of CLCs at ultrahigh frequencies. The problems considered in this

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paper are also interesting from the theoretical point of view.

METHOD OF ADDITION OF LAYERS

Let a wave $\mathbf{E}_i$ be incident from the left on a plane layer giving rise to the waves $\mathbf{E}_r$ and $\mathbf{E}_t$ reflected and transmitted through the layer, respectively. Let us expand the complex amplitudes of the incident, reflected, and transmitted waves in the circular base polarizations

$$\mathbf{E}_{i,r,t} = E^{+}_{i,r,t}\mathbf{n}_+ + E^{-}_{i,r,t}\mathbf{n}_- = \begin{bmatrix} E^{+}_{i,r,t} \\ E^{-}_{i,r,t} \end{bmatrix}, \quad (1)$$

where $\mathbf{n}_+$, $\mathbf{n}_-$ are the unit vectors of the circular base polarizations. The reflected and transmitted waves are related to the incident wave by the expressions

$$\mathbf{E}_r = \hat{R}\mathbf{E}_i, \quad \mathbf{E}_t = \hat{T}\mathbf{E}_i, \quad (2)$$

where $\hat{R}$ and $\mathbf{T}$ are the Jones matrices for the given layer.

Consider now a system consisting of two layers A and B adjoined from "left to right" to each other. Then, similar to [9], it can be easily shown that upon incidence of the wave on the composite $A + B$ layer, the reflection, $\hat{R}_{A+B}$, and transmission, $\hat{T}_{A+B}$, matrices are expressed in terms of matrices A and B of the corresponding layers in the form:

$$\begin{aligned} \hat{R}_{A+B} &= \hat{R}_A + \hat{\tilde{T}}_A\hat{S}\hat{T}_A, \\ \hat{T}_{A+B} &= \hat{T}_B\hat{P}\hat{T}_A. \end{aligned} \quad (3)$$

The amplitudes $\hat{S}$ and $\hat{P}$ describe the resulting waves that arise at the interface of the layers A and B when this "sewing" plane itself represents a primary radiation source. The advantage of the use of matrices $\hat{S}$ and $\hat{P}$ is, in particular, that in this case a complication of the problem (addition of radiating plane sources between the layers or inside them or the passage from the study of parameters of reflection and transmission to analysis of parameters of the internal fields in the optical system) does not lead to a necessity for solving new equations. This is discussed in more detail in [9, 10, 12], while our treatment below is restricted by the reflection–transmission problem.

The sewing matrices $\hat{S}$ and $\hat{P}$ can be found from the system [9]

$$\begin{aligned} \hat{P} &= \hat{I} + \hat{\tilde{R}}_A\hat{S}, \\ \hat{S} &= \hat{R}_B\hat{P}, \end{aligned} \quad (4)$$

where $\hat{I}$ is the unit matrix, the tilde denotes the reflection–transmission properties of the layer when the wave is incident from the right side. For example, when the layer is adjacent to the same medium from both its sides, the Jones matrices for the light incident from the right and left are related by the expressions:

$$\begin{aligned} \hat{\tilde{T}} &= \hat{F}^{-1}\hat{T}\hat{F}, \\ \hat{\tilde{R}} &= \hat{F}^{-1}\hat{R}\hat{F}, \end{aligned} \quad (5)$$

where $\hat{F} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ for circular base polarizations.

By solving system (4), we obtain for $\hat{S}$ and $\hat{P}$

$$\begin{aligned} \hat{S} &= \hat{R}_B[\hat{I} - \hat{\tilde{R}}_A\hat{R}_B]^{-1}, \\ \hat{P} &= [\hat{I} - \hat{\tilde{R}}_A\hat{R}_B]^{-1}. \end{aligned} \quad (6)$$

These addition formulas can be used repeatedly, by choosing the layers A and B according to the conditions of a specific problem (addition formulas are discussed in more detail in [9, 10]). In this way, one can construct recurrently (numerically or analytically) and calculate an optical system with an arbitrary number of sublayers with arbitrary (specified beforehand) characteristics of the components.

LIGHT REFLECTION AND TRANSMISSION BY THE CLC–GLASS–CLC SYSTEM

Let a light wave be incident normally on the CLC–glass–CLC system. We will solve the problem of reflection and transmission of light by this system in two stages: First, a glass layer is joined from the left to the CLC(2) layer, and then the CLC (1) layer is joined again from the left to this system. The glass layer is uniform and optically isotropic and is characterized by a complex refractive index n. This method can also be applied to the description of anisotropic or gyrotropic layers. In this case, the Jones matrices for an isotropic glass plate are replaced by the corresponding matrices for anisotropic or gyrotropic plates.

Let A and B be a glass layer and the second layer of the CLC, respectively. Then, using the analytic expressions for the Jones matrix elements for individual glass layers [8] and CLC layers [4] and expressions (1)–(6), one can obtain the exact analytic expressions for the Jones matrix elements for the glass–CLC(2) system in the form:

$$\begin{aligned} T^a_{jj} &= t\exp[(-1)^{j-1}ia^{(2)}d^{(2)}] \\ &\times[Q^{(2)}_j(1-\tilde{r}H^{(2)}) + V^{(2)}F^{(2)}_j]/\Delta_1, \\ T^a_{jk} &= t\exp[(-1)^{j-1}ia^{(2)}d^{(2)}] \\ &\times[V^{(2)}(1-\tilde{r}H^{(2)}) + Q^{(2)}_jF^{(2)}_k]/\Delta_1, \\ R^a_{jj} &= r + \tilde{t}t[H^{(2)}(1-\tilde{r}H^{(2)}) + F^{(2)2}_j]/\Delta_1, \\ R^a_{jk} &= \tilde{t}t[F^{(2)}_j(1-\tilde{r}H^{(2)}) + H^{(2)}F^{(2)}_k]/\Delta_1, \end{aligned} \quad (7)$$

$$j, k = 1, 2, \quad j \neq k,$$

where

$$\Delta_1 = (1 - \tilde{r}H^{(2)})^2 - F_1^{(2)}F_2^{(2)},$$

$$Q_{1,2}^{(n)} = \gamma\sqrt{\alpha}\{c_1(\gamma\sqrt{\alpha} \mp \chi r_1) + c_2(\gamma\sqrt{\alpha} \pm \chi r_1)$$

$$- iu[s_1(w_1 \mp 2l_1\sqrt{\alpha}) + s_2(w_2 \mp 2l_2\sqrt{\alpha})]\}/\Delta,$$

$$V^{(n)} = \delta\gamma\sqrt{\alpha}[\sqrt{\alpha}(c_2 - c_1) + iu(s_1q_1 - s_2q_2)]/\Delta,$$

$$H^{(n)} = \{\chi^2 r_1 r_2(c_2c_1 - 1)$$

$$+ 2u^2[r_1r_2(2\chi^2 m_1 - \gamma^2) - \gamma^2\delta^2\alpha^2]s_2s_1$$

$$- iu\gamma\sqrt{\alpha}(p_1c_2s_1 + p_2c_1s_2)\}/\Delta,$$

$$F_{1,2}^{(n)} = \delta\sqrt{\alpha}\{4u^2[\sqrt{\alpha}\gamma^2 \pm (m_1r_2 + \gamma^2\alpha)\chi]s_2s_1$$

$$\pm \chi r_2(c_2c_1 - 1) + iu\gamma[(g_1 \pm 2\chi\sqrt{\alpha})c_1s_2$$

$$- (g_2 \pm 2\chi\sqrt{\alpha})c_2s_1]\}/\Delta,$$

$$\Delta = -\chi^2 r_2^2 + (\chi^2 r_2^2 + 2\alpha\gamma^2)c_2c_1$$

$$+ 2u^2[4\chi^2\alpha(\delta^2 - 2m_2) - r_1^2(2\chi^2 m_2 + \delta^2)$$

$$+ \gamma^2\delta^2\alpha^2]s_2s_1 - 2iu\gamma\sqrt{\alpha}(w_1c_2s_1 + w_2c_1s_2),$$

$$b_{1,2} = \gamma \pm 2\chi^2, \quad w_{1,2} = r_1b_{1,2} \pm \delta^2\alpha,$$

$$p_{1,2} = r_2b_{1,2} \mp \delta^2\alpha, \quad g_{1,2} = r_2 \pm \alpha\gamma,$$

$$q_{1,2} = r_1 \pm \alpha\gamma, \quad r_{1,2} = 1 \pm \alpha, \quad m_{1,2} = 1 \pm \chi^2,$$

$$\alpha = \varepsilon_m/\varepsilon_v, \quad \gamma = \sqrt{4\chi^2 + \delta^2}, \quad l_{1,2} = \gamma \pm 2,$$

$$u = \pi d^{(n)}\sqrt{\varepsilon_m}/\lambda, \quad \chi = \lambda/\sigma\sqrt{\varepsilon_m},$$

$$\delta = \varepsilon_a/\varepsilon_m, \quad c_{1,2} = \cos(k_{1,2}d^{(n)}),$$

$$s_{1,2} = \sin(k_{1,2}d^{(n)})/k_{1,2}d^{(n)}, \quad k_{1,2} = 2uf_{1,2}/d^{(n)},$$

$$f_{1,2} = \sqrt{1 + \chi^2 \pm \gamma}, \quad \varepsilon_a = (\varepsilon_1 - \varepsilon_2)/2,$$

$$\varepsilon_m = (\varepsilon_1 + \varepsilon_2)/2,$$

$$r = [\tau_{12} + \tau_{23}\exp(-i2\beta)]/[1 + \tau_{12}\tau_{23}\exp(-i2\beta)],$$

$$\tilde{r} = r, \quad t = t_{12}t_{23}\exp(-i\beta)/[1 + \tau_{12}\tau_{23}\exp(-i2\beta)],$$

$$\tilde{t} = t, \quad \tau_{12} = (n_v - n)/(n_v + n), \quad \tau_{23} = -\tau_{12},$$

$$t_{12} = 2n_v/(n + n_v), \; t_{23} = 2n/(n + n_v), \; \beta = 2\pi nd/\lambda,$$

$\varepsilon_{1,2}$ are the principle values of the dielectric constant tensor of the CLC in the plane perpendicular to the system axes; λ is the wavelength in vacuum; σ is the pitch of helix; $d^{(n)}$ is the thickness of the CLC layer; $a = 2\pi/\sigma$, d is the thickness of the glass plate; n, n_v are refractive indices of a glass plate and vacuum, respectively; ε_v is the dielectric constant of the vacuum; the superscript n ($n = 1, 2$) in parentheses denotes the number of the CLC layer; and the quantities $Q_j^{(n)}$, $V^{(n)}$, $F_j^{(n)}$, and $H^{(n)}$ depend on the corresponding parameters of the nth layer of the CLC (the superscripts of these parameters were omitted for simplicity).

At the second stage, we will join the first CLC layer to the obtained system from the left. Now, we will treat A as the first CLC layer and B, as the glass–CLC(2) system.

By using explicit expressions for the matrix elements of the composite structures, we obtain, with the help of equations (1)–(6), for the elements of the Jones matrix describing the CLC(1)–glass–CLC(2) system

$$T_{ij} = \sum_{k=1}^{2} T_{ik}^a \left(\sum_{m=1}^{2} P_{km} T_{mj}^{(1)} \right),$$

$$R_{ij} = R_{ij}^{(1)} + \sum_{k=1}^{2} T_{3-i,3-k}^{(1)} \left(\sum_{m=1}^{2} S_{km} T_{mj}^{(1)} \right), \quad i, j = 1, 2, \tag{8}$$

where

$$S_{ij} = \sum_{k=1}^{2} R_{ik}^a P_{kj}, \quad i, j = 1, 2,$$

$$P_{ii} = (1 - R_{ii}^{(1)}R_{jj}^a - R_{ij}^{(1)}R_{ij}^a)/\Delta_2,$$

$$P_{ij} = (R_{ij}^{(1)}R_{ii}^a + R_{ii}^{(1)}R_{ji}^a)/\Delta_2,$$

$$R_{ii}^{(1)} = H^{(1)}, \quad R_{ij}^{(1)} = F_i^{(1)},$$

$$T_{ii}^{(1)} = Q_i^{(1)}\exp[-(-1)^i a^{(1)}d^{(1)}],$$

$$T_{ij}^{(1)} = V^{(1)}\exp[-(-1)^i a^{(1)}d^{(1)}], \quad i, j = 1, 2, \quad i \neq j,$$

$$\Delta_2 = (1 - R_{21}^{(1)}R_{21}^a)(1 - R_{12}^{(1)}R_{12}^a)$$

$$- R_{11}^{(1)}R_{22}^a(1 - R_{22}^{(1)}R_{11}^a) - R_{21}^a(R_{21}^{(1)} + R_{11}^{(1)}R_{22}^{(1)}R_{12}^a)$$

$$- R_{11}^a(R_{22}^{(1)} + R_{21}^{(1)}R_{12}^{(1)}R_{22}^a).$$

Figure 1 shows the dependence of the reflection coefficient on the wavelength for the incident light with linear polarization (along the x-axes) (curve *1*), left-hand (curve *2*) and right-hand (curve *3*) circular polarization. The CLC layers differ by the pitch value only. The pitches were chosen to provide the separation of regions of selective reflection. If the isotropic layer absorbs the light of only one circular polarization (left- or right-hand), this system represents a narrow-band filter for right- or left-hand polarization, respectively. Because the width of the transmission band

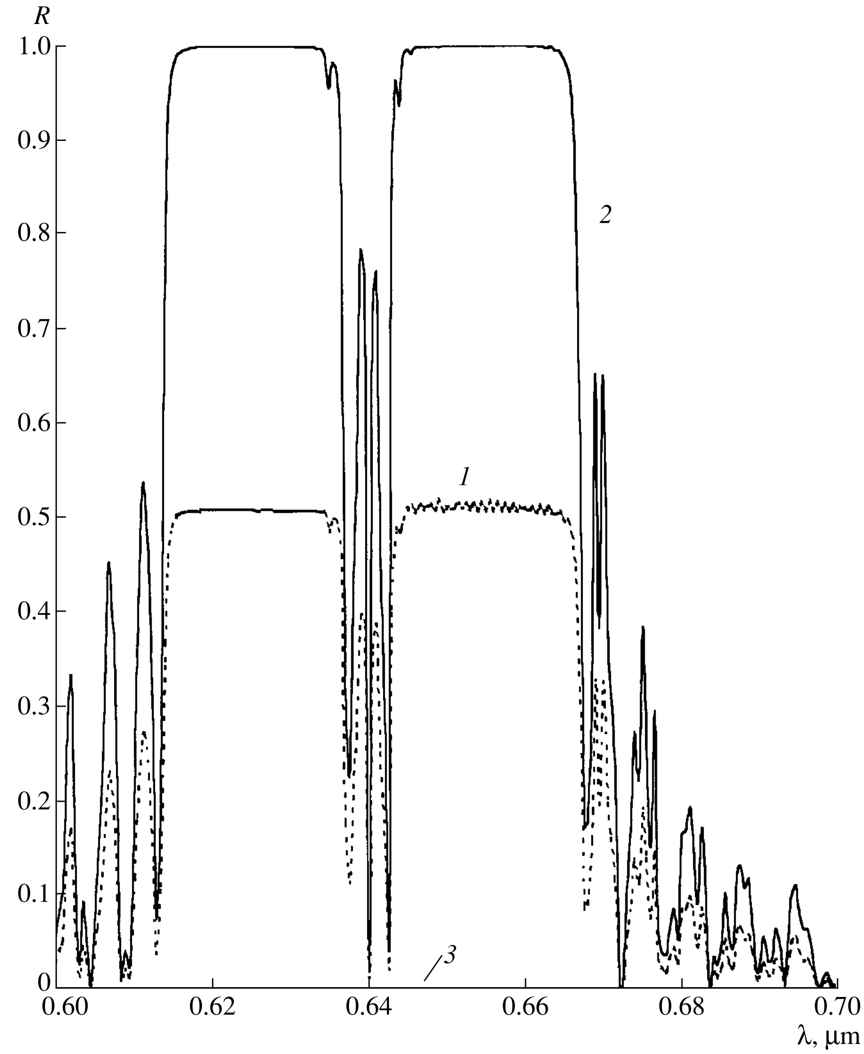

Fig. 1. Dependence of the reflection coefficient on the wavelength of light with (*1*) linear, (*2*) left-hand, and (*3*) right-hand circular polarizations. The parameters of the system are $\varepsilon_1^{(1)} = \varepsilon_1^{(2)} = 2.29$, $\varepsilon_2^{(1)} = \varepsilon_2^{(2)} = 2.143$, $\sigma^{(1)} = 0.42$ μm, $\sigma^{(2)} = 0.44$ μm, $d^{(1,2)} = 50|\sigma^{(1,2)}|$, $d = 1000$ μm, $n = 1.5$.

$\Delta\lambda = (\sigma^{(1)}\sqrt{\varepsilon_m^{(1)}(1+\delta^{(1)})} - \sigma^{(2)}\sqrt{\varepsilon_m^{(2)}(1-\delta^{(2)})}$, $|\sigma^{(2)}| > |\sigma^{(1)}|$, the spectral region of the transmission band can be varied by changing the pitch of the CLC helix by varying temperature or external fields (electric, magnetic, or hypersonic). Thus, we have a narrow-band filter of circular polarization with the controlled bandwidth $\Delta\lambda$ and tunable center frequency. Such a filter can be also manufactured using three CLC layers with appropriately chosen values and signs of the pitch of helixes of layers. Selective filters with similar properties for linear polarization can be composed, for example, from CLC($\sigma^{(1)}$, $d^{(1)}$)–CLC($-\sigma^{(1)}$, $d^{(2)}$)–glass–CLC($\sigma^{(2)}$, $d^{(3)}$)–CLC($-\sigma^{(2)}$, $d^{(4)}$) system.

When the CLC layers have different signs and values of the pitch of helix (and the helix pitches are chosen to obtain slightly overlapped regions of selective reflection), the calculations of the wavelength dependence of the reflection coefficient show that in the overlap region the system reflects any polarization. Therefore, such a system can be used as a tunable narrow-band mirror.

Note again that the advantage of "filters" and "mirrors" under study is their high spectral resolution and the tunability of the transmission and reflection bands, as well as the possibility of varying the widths of these bands.

Although we discussed only three possible applications of the systems described, we can propose a variety of other applications of these systems.

Note that the apparent absence of coherence effects in reflection (effects of multiple reflections) is explained by the fact that we considered CLCs with

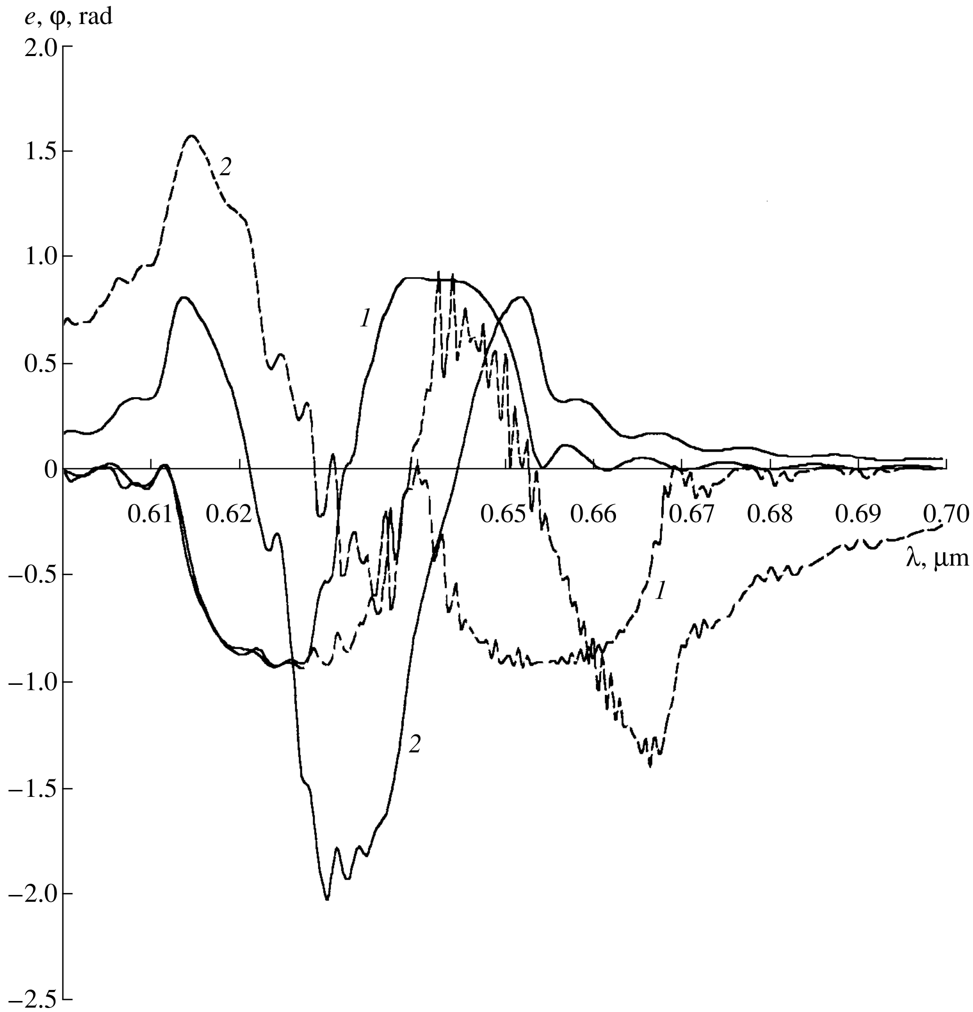

Fig. 2. (*1*) Ellipticity e and (*2*) optical rotation φ as functions of the wavelength of the incident with light linear polarization; $d^{(1,2)} = 35|\sigma^{(1,2)}|$. The dashed curves correspond to the case when the helixes have the same sign, and the solid curves correspond to the opposite signs of helixes. The rest of the parameters are the same as in Fig. 1.

equal values of the average dielectric constant. In addition, the dielectric constant of the isotropic glass was also chosen approximately equal to the average dielectric constant of CLC layers. It is obvious that more detailed numerical calculations will reveal different coherent effects. These effects should be most pronounced in polarization properties of the CLC–glass–CLC system.

Figure 2 shows the dependence of optical rotation φ and the ellipticity e on the wavelength for the two cases considered above. The incident wave has the linear polarization (along the x-axis).

Calculations of the dependence of the reflection coefficient on the wavelength for different refractive indices of the isotropic glass show that when the dielectric constant of the glass substantially differs from the average dielectric constant of the CLC layers, this dependence becomes oscillatory even in the region of selective reflection. This is caused by multiple reflections in the glass plate. Note that multiple reflections substantially change the shape of the diffraction plateau of the CLC layer behind the glass plate.

SPECIAL FEATURES OF EIGENPOLARIZATIONS

It is known that the eigenpolarizations are the two polarizations that do not change after the propagation of light through an optical system. The properties of eigenpolarizations of a layer with periodic helical structure were studied in [13], where it was shown that in the case of a weak local anisotropy of the refractive index, they represent two near-circular polarizations (right- and left-hand). An important feature of periodical helical structures is that they show complete selectivity not with respect to circular polarizations but with respect to eigenpolarizations, which become circular

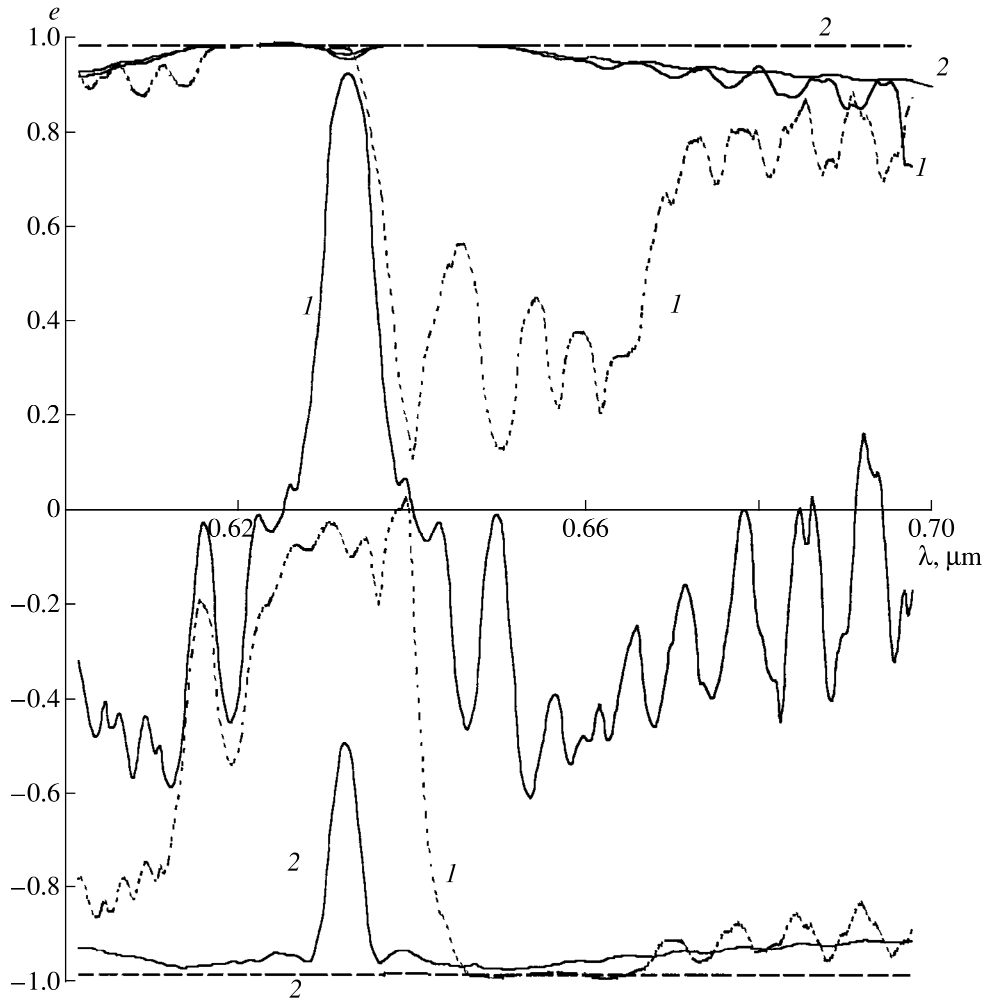

Fig. 3. Dependences of the ellipticity of eigenpolarizations on the wavelength for different refractive indices of the glass plate. The dashed curves correspond to the case when the helixes have the same sign ($\sigma^{(1)} = 0.42$ μm, $\sigma^{(2)} = 0.44$ μm), and the solid lines correspond to the opposite signs of helixes ($\sigma^{(1)} = 0.42$ μm, $\sigma^{(2)} = 0.43$ μm). (*1*) correspond to $n = 0.5$, (*2*) to $n = 1.5$. Other parameters are the same as in Fig. 1.

only in the limit $\delta \ll 1$. The light with one of the eigenpolarizations undergoes diffraction reflection, whereas the light with the other eigenpolarization is not reflected. Analysis of the influence of the anisotropy δ on the eigenpolarizations of the CLC layer showed that their ellipticity decreases (in modulus) with increasing δ, (by modulus) and in the limiting case $\delta \gg 1$, they are transformed to orthogonal linear polarizations

Note also that, while in the absence of absorption the eigenpolarizations are orthogonal, in the presence of absorption, they cease to be orthogonal. In the presence of absorption, the eigenpolarizations of usual gyrotropic media are also nonorthogonal [14]. They cease to be orthogonal in the presence of dielectric interfaces, i.e., when the dielectric constant of the CLC adjacent to the layer differs from the average dielectric constant of the CLC. In this case, the ellipticity of the eigenpolarizations has the same modulus but opposite signs.

The eigenpolarizations of the CLC–glass–CLC system have the following properties.

In the case of weak local anisotropy of refractive indices of the CLC layers and a small difference between the dielectric constant of a glass and the average dielectric constant of the CLC, the eigenpolarizations are approximately circular (left- and right-hand). Unlike the case of a single CLC layer, these polarizations are not orthogonal even in the absence of absorption, and the ellipticity of eigenpolarizations also differ even in the modulus. If the signs of the helixes of the layers are the same, the light with one of the eigenpolarizations undergoes the diffraction reflection from the layer, whereas the light with another eigenpolarization is not reflected. If the helix signs are different, the light with one of the eigenpolarizations is reflected from one

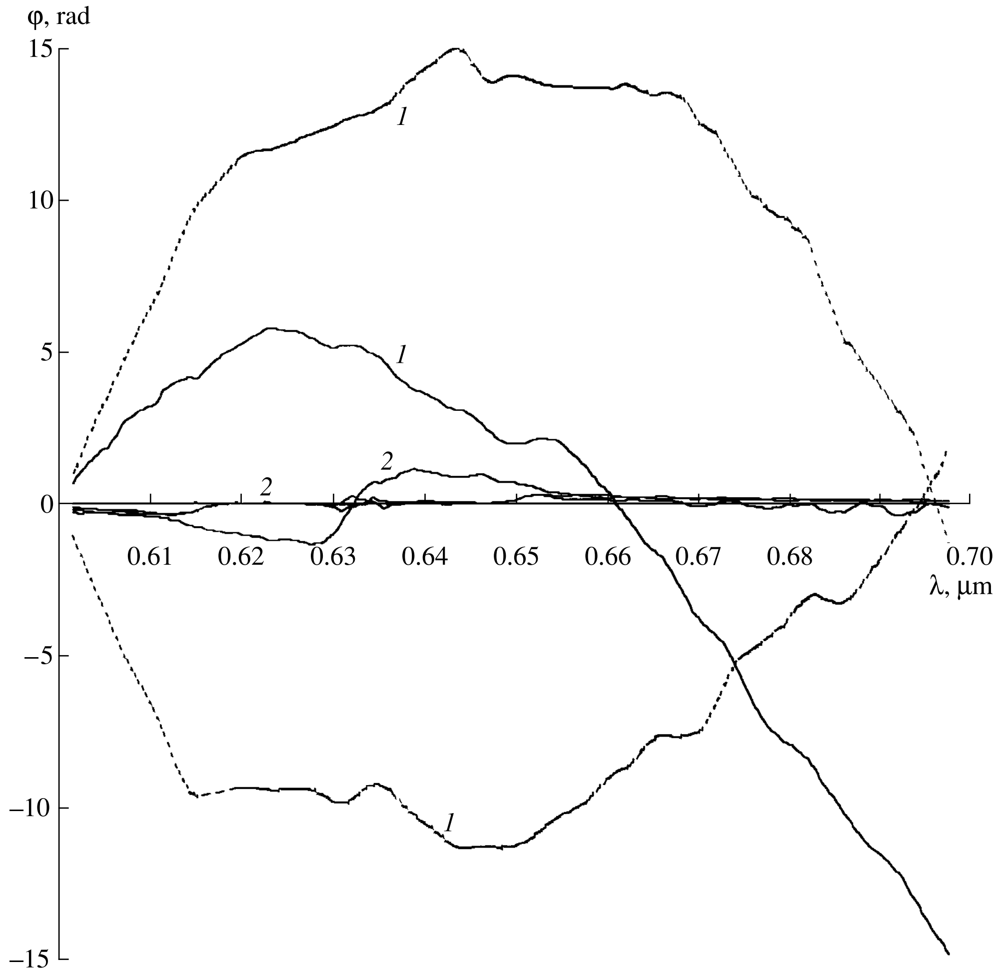

Fig. 4. Dependences of the azimuth of eigenpolarizations on the wavelength for different refractive indices of the glass late. Numbering and parameters are the same as in Fig. 3.

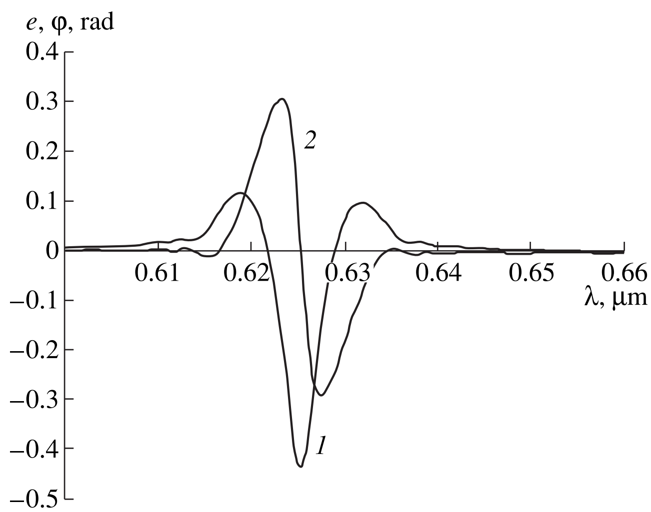

Fig. 5. Dependences of the (*1*) ellipticity and (*2*) rotation on the wavelength. The incident wave has linear polarization. The CLC layers differ only in the helix sign. $\sigma^{(1,2)} = 0.42\ \mu\text{m}$, $d^{(1,2)} = 35\sigma^{(1,2)}$, $d = 0$. Other parameters are the same as in Fig. 1.

of the layers, whereas the light with another eigenpolarization is reflected by the second layer.

The eigenpolarizations depend substantially on the dielectric constant of the isotropic glass. Figures 3 and 4 show the dependences of the ellipticity and azimuth of the eigenpolarizations on refractive index of the glass plate. The dashed curves correspond to the case when the signs of helixes are the same, and the solid curves correspond to the case of the opposite signs.

REFLECTION AND TRANSMISSION OF THE LIGHT BY SYSTEM CLC(1)–CLC(2)

The solution of this problem can be obtained by substitution $d = 0$ in (8) or directly by sewing the first CLC layer with the second one. In the general case, the explicit expressions for the elements of the Jones matrices are cumbersome. More simple expressions are obtained for the case when the CLC layers differ only by the sign of the helix and the thickness of the layers satisfy the conditions: $a^{(n)}d^{(n)} = 2\pi j$, $j = 1, 2, \ldots$, i.e., when the widths of the layers are a multiple of the helix pitch and $\alpha^{(n)} = 1$, $n = 1, 2$. In this case, the elements of the Jones matrix have the form:

$$T_{ii} = \{\delta^2(\xi^2+1) + 8\chi^2\xi$$
$$+ u^2\delta^2[\delta^2(v_2 - \xi v_1)^2 + 2\xi\gamma^2 v_1 v_2]\}/(\Delta\Delta_1\xi\gamma),$$
$$T_{ij} = \delta[h_j(1-\xi)$$
$$+ u^2\delta^2(h_j v_2 + \xi h_i v_1)(v_2 - \xi v_1)]/(\Delta\Delta_1\xi\gamma),$$
$$R_{ii} = iu\delta^2\{(v_1 - v_2) + 2[(v_1 - v_2)(\gamma^2 + 4\chi^2)$$
$$+ 2\chi(\xi^2 v_1 - v_2)(2\chi + \gamma G)/\xi$$
$$+ \gamma G(h_1 v_2 + h_2 v_1)]/(\Delta\Delta_1\gamma)\}/(2\gamma),$$
$$R_{ij} = iu\delta\{(v_1 h_j + v_2 h_i)\Delta_1$$
$$+ \{(3+G)[\delta^2(v_1 h_j + \xi^2 v_2 h_i) + 4(-1)^j\chi\xi(v_1 - v_2)]$$
$$+ 4\chi^2(v_2 h_j + \xi^2 v_1 h_i)] + \gamma^2(1+3G)$$
$$\times(v_2 h_j + \xi^2 v_1 h_i)\}/(2\Delta\xi\gamma)\}/(2\Delta_1\gamma),$$
$$i, j = 1, 2, \quad i \neq j,$$

where $G = u^2\delta^2 v_2 v_1$, $\xi = (c_1 - iul_1s_1)/(c_2 + iul_2s_2)$, $h_{1,2} = \gamma \pm 2\chi$, $v_{1,2} = s_{1,2}/(c_{1,2} \mp iul_{1,2}s_{1,2})$, $\Delta_1 = (1+G)^2 + u^2\delta^4(v_2 - v_1)^2/\gamma^2$, $\Delta = 2\gamma s_1 s_2/v_1 v_2$.

The calculations showed that this system exhibits the properties of an ideal mirror: In the region of selective reflection, the light of any polarization undergoes 100% reflection.

Figure 5 shows the dependences of the optical rotation φ and the polarization ellipticity e on the wavelength. One can see that in the region of diffraction reflection, φ differs substantially from zero. The optical rotation also differs from zero outside this region, but only slightly. The nonzero optical rotation is caused both by multiple reflections and the dependence of the rotation in each layer on the azimuth and the ellipticity of polarization of the incident light.

The study of properties of the eigenpolarizations of this system shows that they are degenerate; i.e., the eigenpolarizations coincide. This degeneracy does not disappear when an isotropic glass plate is introduced between the CLC layers.

In conclusion, note that the dependences obtained can be tested in real experiments and used for the development of a variety of optical elements based on the multilayer CLC structures.

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